GRAVITY ON A DILATONIC GAUSS-BONNET BRANE WORLD

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The effective four-dimensional, linearised gravity of a Randall-Sundrum-like brane world model is analysed [1]. The model includes higher order curvature terms (such as the Gauss-Bonnet term) and a scalar field. The resulting brane worlds can have better agreement with observations than the equivalent Einstein gravity models.

1 Brane Worlds and Higher Order Gravity

In the second Randall-Sundrum (RS) brane world scenario [2], we live on 3+1 dimensional brane embedded in a 4+1 dimensional bulk spacetime. As a result of the warping of the fifth dimension, the effective gravitational theory on the brane closely resembles that which is observed in our universe (except at very small distances). In this paper we will investigate an extended version of this scenario, which has higher order gravity and a scalar field, ϕ, in the bulk. We will consider Z_2-symmetric solutions of the form $ds^2 = e^{-2k|z|}dx_4^2 + dz^2$ and $\phi = -\sigma|z|$ which is the simplest generalisation of the RS model (in some sense σ is the scalar field equivalent of the warp factor k). To avoid bulk singularities σ needs to be positive.

In four dimensions, the gravitational field equations (for the vacuum) are taken to be $G_{ab} + \Lambda g_{ab} = 0$. These can be derived by looking for a rank 2 curvature tensor which (i) is symmetric, (ii) is divergence free, and (iii) depends only on the metric and its first two derivatives. In five dimensions the above conditions are satisfied by $G_{ab} + 2\alpha H_{ab} + \Lambda g_{ab} = 0$, where H_{ab} is the second order Lovelock tensor [3]. H_{ab} can be obtained from the variation of an action containing the Gauss-Bonnet term

$$\mathcal{L}_{\rm GB} = R^{abcd}R_{abcd} - 4R_{ab}R^{ab} + R^2 \ . \tag{1}$$

Energy momentum is conserved in the corresponding gravitational theory and its vacuum is ghost-free (just as in Einstein gravity). Note that H_{ab} is the only quadratic curvature term which satisfies the above three conditions. In four dimensions its contribution to the field equations is trivial, and so it is usually ignored.

The Gauss-Bonnet term also appears in low energy effective actions derived from string theory. Since the brane worlds are loosely motivated by string theory, it is particularly natural for them to include higher gravity terms. String theory also suggests that the bulk space will contain not only gravity, but many other fields. One such field is the dilaton. For simplicity we will include only include this one extra bulk scalar field.

If a scalar field is present, it is natural to include higher order scalar kinetic terms as well as the higher order curvature terms. We will consider the general second order contribution to the action (in the string frame)

$$\mathcal{L}_2 = c_1\mathcal{L}_{\rm GB} - 16c_2 G_{ab}\nabla^a\phi\nabla^b\phi + 16c_3(\nabla\phi)^2\nabla^2\phi - 16c_4(\nabla\phi)^4 \ . \tag{2}$$

For simplicity we will not consider higher than second order terms. In this case the full bulk action is

$$S_{\rm Bulk} = \frac{1}{2}\int d^5x\sqrt{-g}e^{-2\phi}\left\{R - 4\omega(\nabla\phi)^2 + \mathcal{L}_2 - 2\Lambda\right\} \ . \tag{3}$$

The coefficients can be determined from origin of ϕ. We will take $\omega = -1$ and $c_i = \alpha$ which corresponds to the dilaton (with some extra symmetries).

The brane can be treated as a boundary of the bulk spacetime. In this case we need to add a Gibbons-Hawking boundary term (and corresponding higher curvature terms) to the action

$$S_{\rm brane} = -\int d^4x\sqrt{-h}e^{-2\phi}\left\{2K + \mathcal{L}_2^{(b)} + T\right\} \tag{4}$$

$$\mathcal{L}_2^{(b)} = c_1\mathcal{L}_{\rm GB}^{(b)} - 16c_2(K_{ab} - Kh_{ab})D^a\phi D^b\phi - 16c_3(n\cdot\nabla\phi)\left(\frac{1}{3}(n\cdot\nabla\phi)^2 + (D\phi)^2\right) \tag{5}$$

where [4]

$$\mathcal{L}_{\rm GB}^{(b)} = \frac{4}{3}(3KK_{ac}K^{ac} - 2K_{ac}K^{cb}K^a{}_b - K^3) - 8G_{ab}^{(4)}K^{ab} \ . \tag{6}$$

Variation of the action gives the generalised Israel junction conditions for the brane[5]. These do not depend on the brane thickness (this is not true for other second order gravity terms).

For the type of solutions we are considering, there are three different solution branches for the bulk field equations

$$\text{(a)} \quad k = 0 \quad , \quad \text{(b)} \quad k = \sigma - \sqrt{\frac{1}{12\alpha} + \frac{\sigma^2}{3}} \quad , \quad \text{(c)} \quad k = \sigma + \sqrt{\frac{1}{12\alpha} + \frac{\sigma^2}{3}} \ . \tag{7}$$

The first is always valid, while the other two are only possible when the higher order terms (2) are included in the action.

2 Linearised Brane World Gravity

When analysing the effective four-dimensional brane gravity we need to worry about perturbations of the brane position as well as bulk metric. This can be addressed by also perturbing the coordinates, or by using gauge in which brane stays at $z = 0$ [6] (the approach we will use).

Consider a general perturbation of the RS-like brane world solutions with

$$ds^2 = e^{-2k|z|}(\eta_{\mu\nu} + \gamma_{\mu\nu})dx^\mu dx^\nu + 2v_\mu dx^\mu dz + (1+\psi)dz^2 \tag{8}$$

and $\phi = -\sigma|z| + \varphi$, where $\gamma_{\mu\nu}$, v_μ, ψ and φ are small. It is useful to split $\gamma_{\mu\nu}$ into tensor and scalar parts

$$\gamma_{\mu\nu} = \bar{\gamma}_{\mu\nu} + \frac{1}{4}\gamma\eta_{\mu\nu} + \frac{4}{3}c_\chi\left(\frac{1}{4}\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box_4}\right)\chi \ , \tag{9}$$

where $\gamma = \eta^{\mu\nu}\gamma_{\mu\nu}$ and $\partial^\mu\bar{\gamma}_{\mu\nu} = 0$. The field χ will be some linear combination of γ and φ. If the brane remains at $z = 0$, the bulk field equations are satisfied by $\psi = 2\partial_z(\chi - \varphi)/\sigma$ and $v_\mu = \partial_\mu(\chi - \varphi)/\sigma$.

3 New Instabilities

The graviton wave equation is obtained from the remaining bulk field equations.

$$\mu_\gamma \left(\partial_z^2 - 2(2k-\sigma)\partial_z + f_\gamma^2 e^{2kz}\Box_4\right)\bar{\gamma}_{\mu\nu} = 0 \tag{10}$$

where $\mu_\gamma = 1 - 4\alpha[k^2 - 4k\sigma + 2\sigma^2]$ and $f_\gamma^2 = 1 - 8\alpha\sigma k/\mu_\gamma$. If either of μ_γ or f_γ^2 is negative, the kinetic term in the effective bulk action for $\bar{\gamma}_{\mu\nu}$ will have the wrong sign, and so the theory will have ghosts [7]. This is not possible if $\alpha = 0$ (i.e. if the higher order gravity terms are absent).

The brane junction conditions imply

$$\mu_\gamma\partial_z\bar{\gamma}_{\mu\nu} + 4\alpha[k-2\sigma]\Box_4\bar{\gamma}_{\mu\nu} = -\left\{S_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box_4}\right)S\right\} \tag{11}$$

where $S_{\mu\nu}$ is the brane energy momentum tensor.

If $4\alpha[k-2\sigma] < 0$, then either the effective four-dimensional Planck mass is negative, or the vacuum has non-trivial solutions with spacelike momenta (i.e. tachyons, which implies the solution is unstable). This occurs for the $k=0$ solution branch (7a) if $\alpha > 0$. The model would be stable if $\alpha = 0$ (but it does not give the correct gravitational laws). So in this case the higher order corrections have destabilised the solution. Scalar ghosts and tachyons are also possible in this type of model.

4 New 'Gauss-Bonnet' solutions

When $\alpha > 0$ two new solution branches appear with $k = \sigma \pm \sqrt{1/[12\alpha] + \sigma^2/3}$. The graviton equations are eqs. (10) and (11), and $\mu_\gamma = 8\alpha k(k-\sigma)$ and $f_\gamma^2 = (k-2\sigma)/(k-\sigma)$. Ghosts and tachyons are possible, and we find that the second solution branch (7b) is always unstable. For the rest of this article we will only consider the third solution branch (7c), which is stable if $\sigma < 1/\sqrt{8\alpha}$.

Switching to Fourier space, the bulk graviton equation is solved by

$$\bar{\gamma}_{\mu\nu} \propto e^{(2k-\sigma)z} K_{2-\sigma/k}\left(f_\gamma p e^{kz}/k\right) \tag{12}$$

for spacelike perturbations ($p = \sqrt{p^\mu p_\mu}$).

For $p \ll k/f_\gamma$ (which corresponds to larger distances)

$$\partial_z\bar{\gamma}_{\mu\nu} \approx \frac{f_\gamma^2}{2(k-\sigma)}\Box_4\bar{\gamma}_{\mu\nu} \tag{13}$$

so on the brane, the junction condition (11) reduces to the usual, four-dimensional, linearised Einstein equation at large distances (just as in the RS model). The extra $\Box_4\bar{\gamma}_{\mu\nu}$ term in the junction condition gives four-dimensional gravity at short distances too (unlike the RS model). This significantly weakens the constraints on the model from gravity experiments [8].

Unlike the RS model, we also have a scalar field to worry about. The effective bulk scalar field is

$$\chi = \frac{6\alpha k(k-\sigma)}{2\sigma + 3k(1+4\alpha k^2)}(8k\varphi - \sigma\gamma)\ . \tag{14}$$

The constant c_χ in eq. (9) is then $-\sigma(3k-2\sigma)/[k(k-\sigma)]$.

The bulk equation for χ is qualitatively similar to that of the graviton, but with $\mu_\chi = 16\alpha k(3k-2\sigma)$ and $f_\chi^2 = 3k/(3k-2\sigma)$.

The remaining junction conditions are

$$(3k-2\sigma)[12k^2(k-\sigma) + \alpha(3k-2\sigma)]\partial_z\chi + 12k^2\sigma\Box_4\chi = \frac{S}{16} \tag{15}$$

$$- 4\alpha\sigma k(3k - 2\sigma)\partial_z\chi + 12\alpha k(k - \sigma)\Box_4\chi = \Box_4\varphi \ . \tag{16}$$

These are all qualitatively similar to the graviton equations. Again we find there are no scalar ghosts or tachyons for solution (c) if $\sigma < 1/\sqrt{8\alpha}$. The scalar perturbations give approximately four-dimensional brane gravity at large and small distances, in the same way that the graviton perturbations do. However all the coefficients in the above equations are different. We see that the degeneracy between the behaviours of scalar and tensor modes has broken. In particular we can have $f_\gamma \ll f_\chi$ if σ is near $1/\sqrt{8\alpha}$, and so the two types of perturbation 'feel' the effects of the bulk differently.

Using the solutions of the above field equations, and taking appropriate series and asymptotic expansions, we obtain (to leading order) linearised Brans-Dicke gravity on brane.

$$\mathcal{G}_{\mu\nu} - 2(\eta_{\mu\nu}\Box_4 - \partial_\mu\partial_\nu)\tilde{\varphi} \approx M_{\rm Pl}^{-2} S_{\mu\nu} \ , \quad -2\Box_4\tilde{\varphi} \approx M_\phi^{-2} S \ , \tag{17}$$

where $\mathcal{G}_{\mu\nu}$ is the linearised Einstein tensor corresponding to $\gamma_{\mu\nu}$ and $\tilde{\varphi}(\gamma, \varphi)$ is the effective four-dimensional scalar. The effective (distance dependant) coupling strengths of gravity and the scalar are respectively $M_{\rm Pl}^{-2}$ and M_ϕ^{-2}.

If $p \ll k/f_\chi$ (large distances), we find

$$M_{\rm Pl}^2 = 8\alpha f_\gamma^2 (2k - \sigma) \ , \quad M_\phi^2 = 8\alpha(3k - 2\sigma) \ , \tag{18}$$

so we can have $M_\phi \gg M_{\rm Pl}$ if the solution is fine-tuned to have $f_\gamma \ll 1$. Hence we can potentially avoid conflict with solar system constraints [9]. Note that this is possible despite the fact that the coupling strengths of gravity and ϕ are the same in the underlying five-dimensional theory.

At medium ($k/f_\gamma \gg p \gg k/f_\chi$) and short ($p \gg k/f_\gamma$) distance scales, we find $M_\phi^2 \leq 3M_{\rm Pl}^2$. But this is not a problem, since short distance constraints on Brans-Dicke gravity are weak.

It is interesting to note that the series expansion (13) used to determine the effective large distance gravity is not valid if $\sigma > k$. If $2k > \sigma > k > 0$ we find $\partial_z \bar{\gamma}_{\mu\nu} \sim p^{4-2\sigma/k} \bar{\gamma}_{\mu\nu}$ instead. The corresponding large distance Newton potential will then have a non-standard $r^{1-2\sigma/k}$ behaviour. This is possible for the second solution branch (7b) when $\alpha < 0$. Unfortunately it is has bulk ghosts and so is unstable. It may be possible that by using a different choice of coefficients in eq. (2), a stable version of this type of solution can be found.

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References

1. S. C. Davis, hep-th/0402151.
2. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999) [hep-th/9906064].
3. D. Lovelock, *J. Math. Phys.* **12**, 498 (1971).
4. R. C. Myers, *Phys. Rev.* D **36**, 392 (1987).
5. S. C. Davis, *Phys. Rev.* D **67**, 024030 (2003) [hep-th/0208205].
   E. Gravanis and S. Willison, *Phys. Lett.* B **562**, 118 (2003) [hep-th/0209076].
6. I. Y. Aref'eva, M. G. Ivanov, W. Muck, K. S. Viswanathan and I. V. Volovich, *Nucl. Phys.* B **590**, 273 (2000) [hep-th/0004114];
   H. Collins and B. Holdom, *Phys. Rev.* D **62**, 124008 (2000) [hep-th/0006158].
7. C. Charmousis, S. C. Davis and J. F. Dufaux, *JHEP* **0312**, 029 (2003) [hep-th/0309083].
8. N. Deruelle and M. Sasaki, *Prog. Theor. Phys.* **110**, 441 (2003) [gr-qc/0306032].
9. T. Damour and K. Nordtvedt, *Phys. Rev.* D **48**, 3436 (1993), (and references therein).